

MODMAD Research Federative Structure of the University of Saint-Etienne and ENISE (Engineer School of Saint-Etienne)

International Workshop on the Multiscale Modelling and Methods (MMM)

October 9-11, 2011,
Salle du Conseil - Faculty of Sciences
23 rue du Docteur Paul Michelon - 42000 Saint-Etienne

Organiser: Grigory Panasenko

Invited speakers: V.Chiado-Piat, Italy, Torino, M.Chipot, Switzerland, Zurich, A.Gusarov, France, Saint-Etienne, P.Mironescu, France, Lyon, K.Pankrashkin, France, Paris, I.Pankratova, Norway, Narvik, A.Piatnitski, Norway, Narvik, E. Sanchez-Palencia, France, Paris

The workshop is devoted to a recent and important trend in mathematical modelling : the multiscale methods. Such methods combine the microscopic and macroscopic description of the phenomena and are usually based on the asymptotic and the numerical analysis of the equations of the microscopic model. Working languages: English and French

Program :

Monday, 10/10/2011

12.15 : Opening

14.00-14.30 : M.Chipot (University of Zurich) On the asymptotic behaviour of variational inequalities set in cylinders. The generalized cylinders of the form $L \times w_1 \times w_2$ are considered where w_1 and w_2 are some bounded open sets in \mathbb{R}^p and \mathbb{R}^{n-p} respectively. An asymptotic behaviour in L of the solutions to elliptic variational inequalities set in these cylinders is studied.

14.30-15.00 : K.Pankrashkin (University Paris-Sud 11) Réduction dimensionnelle et équivalence unitaire pour une classe d'extensions autoadjointes avec applications aux opérateurs différentiels sur réseaux. Les opérateurs différentiels sur réseaux apparaissent de façon naturelle dans les études de la propagation des ondes au sein d'un objet dont l'épaisseur est négligeable par rapport à leur longueur. Un réseau est un objet qu'on obtient en remplaçant les liens d'un graphe par segments. En considérant un opérateur différentiel sur chaque lien et en imposant certaines conditions au bord à chaque sommet on obtient un opérateur autoadjoint. Les opérateurs de ce type font l'objet de nombreux travaux en mécanique quantique, chimie quantique, théorie de la supraconductivité etc. (souvent sous le nom "graphe quantique"). Le but de l'exposé est de discuter certaines relations entre les spectres des réseaux et certaines opérateurs aux différences finies. En utilisant les techniques d'extensions autoadjointes on démontre que dans certains cas le problème spectral sur réseaux admet une réduction unitaire au problème spectral pour les laplaciens discrets.

- K. Pankrashkin: Unitary dimension reduction for a class of self-adjoint extensions with applications to graph-like structures. Preprint arXiv:1109.0712.

- J. Brüning, V. Geyler, K. Pankrashkin: Spectra of self-adjoint extensions and applications to solvable Schrödinger operators. Rev. Math. Phys. 20 (2008) 1-70.

- K. Pankrashkin: Spectra of Schrödinger operators on equilateral quantum graphs. Lett. Math. Phys. 77 (2006) 139-154.

15.00-15.30 P.Mironescu (University of Lyon) Ginzburg-Landau minimizers with prescribed degrees and domains size We discuss the minimization of the reduced Ginzburg-Landau energy $\frac{1}{2} \int |\nabla u|^2 + \frac{1}{4\varepsilon^2} (1-|u|^2)^2$, in a 2D multiply connected domain, with prescribed winding numbers on the components of the boundary. This is a critical problem, in which a minimizer may or may not exist. We present several results suggesting the following fact: existence is governed by an appropriate notion of size ; more specifically, minimizers do exist and are unique in thin domains, and do not exist in thick domains.

15.30-16.00 Coffee-break

16.00-16.30 R.Fares (University of Lyon) The Stokes flow in a tube structure with mixed boundary conditions "rigid wall- elastic wall" .

A 2D tube structure constituted of two glued channels is considered. Its thickness is a small parameter. The fluid - structure interaction is modelled by the Sophie Germain type equation (as in [1]) for the elastic part of the wall and non-slip equation for the rigid part. An asymptotic expansion of the solution is constructed and justified.

1. Panasenko G., Stavre R. " Asymptotic analysis of a non-periodic flow in a thin channel with visco-elastic wall", Networks and Heterogeneous Media, 3, 2, 2008, pp. 651-673

16.30-17.00 P.Kurbatova (University of Lyon) The hybrid discrete-continuous models: Asymptotic and numerical study;

The hybrid models in biology are considered. A simplest model, the diffusion -discrete absorption equation is solved asymptotically and the closeness of the hybrid and continuous models. The numerical experiments are discussed [1]

1. P.Kurbatova, G.Panasenko, V.Volpert Asymptotic-numerical analysis of the diffusion- discrete absorption equation, Math. Methods in the Applied Sciences, to appear

17.00-17.30 I.Pankratova (Technical University of Narvik) Localization effect for a spectral problem in a perforated domain with Fourier boundary conditions.

Tuesday 11/10/2011

8.30-9.00 E. Sanchez-Palencia (Institut de France PARIS) "Competition of subspecies and structural stability. Survival of the best adapted or coexistence ?"

Abstract: "We discuss darwinian paradigm using simple models of dynamical systems and structural stability. It appears that complex differentiation leads rather to preservation of diversity than to survival of best adapted".

9.00-9.30 V. Chiado-Piat (University of Turin) Non convex homogenization problems for singular structures

We deal with homogenization problems for non-convex integral functionals depending on vector-valued functions, defined on Sobolev spaces with respect to oscillating measures. Due to non convexity, the approach of two-scale convergence for periodic measure, introduced by Zhikov [1], cannot be applied. So, we combine the localisation method of Γ -convergence, with a discretisation argument which allows to link the oscillating energies to functionals defined on a single Lebesgue space. Moreover, we assume a "discrete" p-connectedness property for the underlying measure, that is intermediate between the ones previously studied by Zhikov [2], and by Bouchitté, Buttazzo, Fragalà [3].

1. Zhikov, V. V. On an extension and an application of the two-scale convergence method. (Russian), Mat. Sb. 191 (2000), 31--72; translation in Sb. Math. 191 (2000), 973--1014

2. V. V. Zhikov, Connectedness and homogenization. Examples of fractal conductivity, Mat. Sbornik 187 (1996), 3-40

3. G. Bouchitté, I. Fragalà. Homogenization of thin structures by two-scale method with respect to measures. SIAM J. Math. Anal. 32 (2001), 1198--1226.

9.30-10.00 A.Piatnitski (Technical University of Narvik) Localization effect for a spectral problem in a perforated domain with Fourier boundary conditions.

10.00-10.30 A.Gusarov (ENISE, Saint-Etienne) Physical models for radiation transfer in multiphase heterogeneous media

Radiation in a representative volume containing a great number of uniform phase domains is characterised by a set of values of its intensity averaged over each phase. A set of radiation transfer equations is derived for the averaged intensities in the framework of the ray optics approach. The equations are validated by comparison of their numerical and analytical solutions with the results obtained by ray tracing. The equations are analysed by expansion in series of eigenfunctions of the scattering operator to obtain the effective radiative thermal conductivity. The results are applied to radiation transfer and radiative heat transfer in packed beds.

10.30-11.00 Coffee break

11.00-11.30 M.C. Viallon (University of Lyon) The finite volume implementation for the asymptotic partial domain decomposition in tube structure.

A one bundle tube structure is considered. First we apply the method of asymptotic partial decomposition of the domain and reduce it to a partially decomposed model of the hybrid dimension [1]. Then the finite volume method is implemented to this hybrid model. The difference between the exact solution and the numerical solution to the finite volume scheme for the partially decomposed problem is estimated.

1. G.Panasenko Multi-scale Modelling for Structures and Composites, Springer, Dordrecht, 2005

2. Panasenko G., Viallon M.C. "The finite volume implementation of the partial asymptotic domain decomposition" Applicable Analysis, 87, 12, 2008, pp.1381-1408.

11.30-12.00 L.Paoli (University of Lyon) Asymptotic behavior of a micropolar fluid flow in a thin domain with rough boundary.

Motivated by lubrication problems, we study the behavior of a micropolar fluid flow in a 2D domain which thickness is of order ε . We prove first the existence and uniqueness of a solution for the (P^ε) problem and we establish a priori estimates. Then we use the two-scale convergence technique to pass to the limit and we prove that the limit problem admits a unique solution.

12.00-12.30 M.Bétoué-Etoughe (Ecole Normale de Libreville, Gabon) Partial homogenization of discrete models.

A two-dimensional discrete model is considered. It is replaced by a semi-discrete model. The difference between the solutions of the discrete and the semi-discrete model is estimated [1]

1. Betoue Etoughe M., Panasenko G., "Partial homogenization of discrete models" Applicable Analysis, 87, 12, 2008, pp.1409-1425.