Modeling of flows in a network of thin tubes

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Navier-Stokes equations in thin tube structures may be applied to model blood flow in the arterial system. An asymptotic analysis of the time dependent three dimensional Navier-Stokes equation in a thin tube structure [1,2] leads to two types of equations for the pressure on the graph of the structure: one of them is a well-known Reynolds equation on the graph with Kirchhoff junction conditions at the nodes (it appears at the slow time scale) ; another equation is a new one proposed by G.Panasenko and K.Pileckas in [1,2]. It couples a one-dimensional, nonlocal in time problem on the graph with a heat equation in the cross-section of the tubes.

We investigate new difference schemes for this problem. Stability and convergence are carefully investigated, theoretically and numerically. In addition, numerical results are compared to the direct numerical solution of the full dimension Navier-Stokes equations [3].

Kernels obtained from the heat equation arise in several modeling contexts, like some double porosity models, or viscous flows in networks of thin tubes. We obtain an asymptotic expansion for small times, which we use to build a numerical scheme for approximating the kernels. Convergence of the scheme and relevance of a correction through the asymptotics are proven both analytically and numerically. Finally, we show that this approximation applies to the model on the graph studied in [3]. This research might be useful to reduce the computational complexity in certain hemodynamical problems.

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