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Title : *On positivity properties in Hecke algebras of Coxeter groups*

Abstract : The Iwahori-Hecke algebra of a finite Weyl group is a central object in representation theory, low-dimensional topology, and geometry of Schubert varieties. While it is naturally related to reductive groups, it can be defined starting from any Coxeter group. In 1979, Kazhdan and Lusztig introduced the famous Kazhdan-Lusztig polynomials, which can be defined as the coefficients of a base change matrix in the Hecke algebra of an arbitrary Coxeter group. They conjectured that these polynomials have nonnegative coefficients.

While the positivity of these polynomials was proven in 1980 in the case of finite and affine Weyl groups using the geometry of the flag variety, the general case remained open until the recent work of Elias and Williamson, who proved the positivity in full generality. Their proof involves a remarkable category of graded bimodules over a polynomial ring introduced by Soergel, which can be defined starting from any Coxeter group (and categorifies its Hecke algebra). The Kazhdan-Lusztig polynomials are then interpreted as graded multiplicities in a canonical filtration of these bimodules.

I will recall the definition of the Hecke algebra and Soergel bimodules. I will then explain how this approach and Elias and Williamson's proof can also be used to deduce a generalization of the positivity of Kazhdan-Lusztig polynomials (and the inverse polynomials), conjectured by Dyer. This uses a categorical action of the braid group on the bounded homotopy category of Soergel bimodules. If time allows we will give some conjectures about the complexes of Soergel bimodules associated to elements of the braid group.