Workshop program (titles and abstracts)

Ashot ALEKSIAN

Title : *Exit time for Self-Interacting diffusions*

Abstract : *In this talk we will discuss Self-Interacting diffusions (SID), its basic properties and applications. We also discuss what constitutes the Exit-time problem, why it is important, and for which dynamical systems it was already established. We present the recent results of exit-time problem for a specific case (convex confinement and convex interaction) of SID and how they were established. In the end of the talk we discuss some ideas to generalize this result*

Armand BERNOU

Title : *Beyond the mean-field limit for the McKean-Vlasov particle system : Uniform in time estimates for the cumulants*

Abstract : *We consider the first-order particle system converging to the McKean-Vlasov SDE in the toroidal setting, with a smooth interaction and in the mean-field scaling. Recent use of the master equation associated to the dynamics have allowed to obtain a new proof of uniform in time propagation of chaos, see Delarue-Tse (2021). We will see how those techniques, combined with the Glauber calculus developed by Duerinckx (2019) for the second-order system without diffusion, allow to control, uniformly in time, the cumulants of the system. This provides a fine estimate of the error of the mean-field approximation and a justification for the fluctuations around this limit. This is joint work with Mitia Duerinckx (FNRS).*

Laetitia COLOMBANI

Title : *Propagation of chaos in a network of FitzHugh-Nagumo*

Abstract : *Neurons can be modeled by Stochastic FitzHugh-Nagumo equations. The specificity of these SDE is a cubic term in the drift, which needs us to pay attention. With Pierre Le Bris, we have studied the behavior of a network on N neurons, interacting with each other, when N tends to infinity. We prove an uniform in time propagation of chaos in a mean-field framework, with a coupling method suggested by Eberle (2016). During this talk, I will present this model and the idea of the method.*

Giacomo DI GESÙ

Title : *Metastability in the joint limit of small noise and large volume*

Abstract : *We consider the Allen-Cahn equation on a finite interval perturbed by space-time white noise. Keeping the size of the spatial domain fixed, the dynamics becomes metastable in the limit of vanishing noise. I will review some sharp metastability estimates in this regime and discuss how the long-time behavior is affected if one allows the system size to grow while the noise vanishes. The talk is based on ongoing work with L. Bertini and P. Buttà (Sapienza University).*

Arnaud GUILLIN

Title : *Uniform in time propagation of chaos for some non convex or singular particle systems*

Abstract : *We will present two approaches to propagation of chaos, uniform in time, in cases which have been for long a real difficulty,namely non convex cases or singular interaction. To this end, we will present a probabilistic approach, i.e. coupling, or analytic, derivation of entropy, to consider non convex cases or vortex 2D model.*
Joint work with Durmus-Eberle-Zimmer or Le Bris-Monmarché.

Samuel HERRMANN

Title : *Exact simulation of the first time a diffusion process overcomes a given threshold*

Abstract : *The aim of our study is to propose a new exact simulation method for the first passage time (FPT) of a diffusion process* $(X_t,\, t \geq 0)$*. We shall consider either a continuous diffusion process (in collaboration with Cristina Zucca, University of Turin) either a jump diffusion (in collaboration with Nicolas Massin, University of Valenciennes). We define* τ_L *the first passage time through the level* L *:*

$$\tau_L := \inf\{t \geq 0 : X_t \geq L\}.$$

In order to exactly simulate τ_L*, we cannot use an explicit expression of its density. The classical way to overcome this difficulty is to use efficient algorithms for the simulation of sample paths, like discretization schemes. Such methods permit to obtain approximations of the first-passage times as a by-product.*
For efficiency reasons, it is particularly challenging to simulate directly this hitting time by avoiding to construct the whole paths. The authors introduce a new rejection sampling algorithm which permits to perform an exact simulation of the first-passage time for general one-dimensional diffusion processes. The main ideas are based both on a previous algorithm pointed out by A. Beskos et G. O. Roberts which uses Girsanov's transformation and on properties of Bessel paths. The efficiency of the method is described through theoretical results and numerical examples.

Jean-François JABIR

Title : *Vanishing noise limit of the first collision-time and the first collision-location between two self-stabilizing diffusions*

Abstract : *The present work investigates the asymptotic regime at the zero-noise limit of the first collision times and first collision locations related to a pair of self-interacting diffusions and of their related particle approximation. These features are considered in a peculiar framework where diffusions evolve in a double-wells self-stabilized landscape, and where collisions are only due to the combined action of the Brownian motions driving the diffusions. As the Brownian actions vanish, we establish a Kramers' type law where the first collision time is shown to grow at an exponential rate and that the related collision location is shown to persist at a given point in space. These results are obtained in two distinctive situations : the general multidimensional (where collisions required to be enlarged) and the one-dimensional (where "true collisions" can be directly studied). Similar results are established for related interacting-particle approximations. This is a joint work with Julian Tugaut.*

Aline KURTZMANN

Title : *Introduction to self-interacting diffusions*

Abstract : *In this talk, we will study some self-interacting diffusions. They are solution to SDEs of the form* $dX_t = dB_t - (\nabla V(X_t) + \frac{1}{t}\int_0^t \nabla W(X_t - X_s)ds)dt$. *We will study the asymptotic behavior of the empirical measure of this process. We will explain how to introduce a deterministic ODE approaching the empirical measure. We will also give sufficient conditions for its convergence.*

Mauro MARIANI

Title : *A Variational approach to stability and limits of McKean-Vlasov dynamics*

Abstract : *Interacting particles systems suggest a variational formulation (as opposed to a differential one) of the McKean-Vlasov and other dynamics. This can equivalently be regarded as a non-linear, non-reversible version of the well-known gradient-flow formulation of the heat equation. We use this formulation to prove some stability results in the low temperature regimes for some classes of McKean-Vlasov dynamics under minimal assumptions. Even in the case of linear equations (diffusion processes) we get some new results concerning convergence of models with critical exponents.*

Mario MAURELLI

Title : *Large deviations for singularly interacting diffusions*

Abstract : *We consider a system of mean-field interacting diffusions with singular drift, in the sub-critical case. As the number of particles tends to infinity, we show a large deviation principle for the empirical measure of this system and consequently the convergence of the system to the associated McKean-Vlasov equation. We exploit tools which may have an interest on their own, like an extended version of Varadhan's lemma and a large deviation principle for Gibbs and Gibbs-like measures with singular potentials. Joint work with Jasper Hoeksema, Thomas Holding, Oliver Tse.*

Pierre MONMARCHÉ

Title : *Metastability for an interacting neuron system*

Abstract : *We consider a system of N neurons, whose membrane potentials follow a mean-field interaction dynamics. More precisely, for each neuron, this potential decreases at constant rate and, on the other hand, it is set back to 0 when the neuron emits a spike, which also induces an increase of the potential of all the other neurons. The spikes occur at random times, at a rate lambda(u) which depends on the membrane potential u. When lamda(u) is zero at zero and differentiable then, for all N, the system almost surely stops in finite time, i.e. there is only a finite number of spikes, followed by a deterministic decay of the system toward 0. We will see that, under some conditions, however, the system is metastable, in the sense that the following points hold : 1) the non-linear limit (as N->infinity) system converges to a unique non-zero equilibrium ; 2) the extinction time of a finite system of size N is exponentially large in N ; 3) the average potential of the system fastly approaches a constant positive value, et the exit times from neighborhood of this value converge (as N->infinity) to the exponential law. The proofs rely on coupling techniques. This is a joint work with Eva Löcherbach.*

Greg PAVLIOTIS

Title : *Mean field limits for weakly interacting diffusions : phase transitions, multiscale analysis and inference*

Abstract : *We consider a system of N weakly interacting particles driven by white noise. The mean field limit of this system is described by the (nonlinear and nonlocal) McKean-Vlasov-Fokker-Planck PDE. We present a detailed analysis of continuous and discontinuous phase transitions for the McKean-Vlasov PDE on the torus. We study the combined diffusive/mean-field limit of systems of weakly interacting diffusions with a periodic interaction potential. We show that, in the presence of phase transitions, the two limits do not commute. We then show the equivalence between uniform propagation of chaos, a uniform in N Logarithmic Sobolev inequality, the absence of phase transitions for the mean field limit, and of Gaussian fluctuations around the McKean-Vlasov PDE. Finally, we develop inference methodologies for estimating parameters in the drift of the McKean SDE using either the stochastic gradient descent algorithm or eigenfunction martingale estimators.*

André SCHLICHTING

Title : *Dynamic of mean-field interacting systems : Phase transitions, mountain passes and metastability*

Abstract : *We present results concerning the qualitative and quantitative description of interacting systems with particular emphasis on those possessing a phase transition under the change of relevant system parameters.*

For this, we first discuss and identify different kinds of phase transitions (continuous and discontinuous) for mean-field limits of interacting particle systems on continuous and discrete state spaces.

Since phase transitions are intimately related to the metastability of the stochastic particle system, we show how a suitable mountain pass theorem in the space of probability measures can describe the metastable behavior of the underlying finite particle system.

We also briefly discuss the very related phenomena of phase-separation for the so-called exchange-driven growth model, which is the mean-field of the cluster dynamic in zero-range and related interacting particle systems. For this model we also prove for a suitable choice of interaction kernels dynamic self-similar behavior which prove is based on a connection to a discrete Bessel-type process.

Denis TALAY
Title : *A few open problems (for me)*
Abstract : *In the study of numerical methods dedicated to the approximation of equilibrium distributions, one faces several problems which, to my knowledge, are not solved yet. The aim of the talk is to present some of them, discuss the technical difficulties to be overcome, and suggest some possible approaches :*
1) on long time estimates over the partial derivatives of solutions to parabolic PDEs (in the elliptic and hypoelliptic cases) ;
2) on long time estimates of the density of diffusion processes (in the elliptic and hypoelliptic cases) ;
3) on the long time convergence of McKean–Vlasov processes and associated particle systems ;
4) on dynamical estimation methods for the asymptotic variance in the CLT for the ergodic theorem ;
5) on the comparison between Euler schemes with constant and decreasing step sizes.

Milica TOMASEVIC
Title : *Quantitative particle approximation of nonlinear Fokker-Planck equations with singular kernel*
Abstract : *In this talk, we will study the convergence of the empirical measure of moderately interacting particle systems with singular interaction kernels. After giving the general setting, we will discuss two main results. First, the quantitative convergence of the time marginals of the empirical measure of particle positions towards the solution of the limiting nonlinear Fokker-Planck equation. Second, the well-posedness for the McKean-Vlasov SDE involving such singular kernels and the convergence of the empirical measure towards it (propagation of chaos).*
These results only require very weak regularity on the interaction kernel, including the Biot-Savart kernel, and attractive kernels such as Riesz and Keller-Segel kernels in arbitrary dimension. In particular, this convergence still holds (locally in time) for PDEs exhibiting a blow-up in finite time. The proofs are based on a semigroup approach combined with a fine analysis of the regularity of infinite-dimensional stochastic convolution integrals. This is a joint work with C. Olivera (Campinas) and A. Richard (Centrale Supelec).

Julian TUGAUT
Title : *When metastability meets nonlinearity*
Abstract : *In this talk, we will first make a review about the first main character : the metastability. Then, we will remind the basic ideas and results about the Freidlin-Wentzell theory. Next, we will present the second main character that is the McKean-Vlasov diffusion, a non-linear SDE. We will give some results about exit-time for this diffusion when there is an attraction. Finally, we will present a joint work with Paul-Eric Chaudru de Raynal, Hong Duong, Pierre Monmarché and Milica Tomasevic : the exit-time when there is a repulsion. Let us point out that this work is neither restricted to McKean-Vlasov diffusion nor overdamped diffusion nor conservative diffusion.*

Jeremy WU
Title : *The gradient flow structure of the Landau equation*
Abstract : *The Landau equation is a nonlinear and nonlocal diffusive equation which is significant in kinetic theory. In this talk, I give a description of the flow of the Landau equation as the gradient flow of the Boltzmann entropy under a new bespoke metric. The metric generalises optimal transportation metrics so that the flow of Landau equation can be viewed as the steepest descent of the Boltzmann entropy with respect to this metric. I discuss the rigorous connection between classical weak solutions to the Landau equation and gradient flow solutions to the Landau equation in the sense of Ambrosio, Gigli, and Savaré. This is joint work with José A. Carrillo, Matias G. Delgadino, and Laurent Desvillettes.*