CONVERGENCE RATE OF STRONG APPROXIMATIONS OF COMPOUND RANDOM MAPS and APPLICATIONS .

Mrad Mohamed (LAGA)
Institut Camille Jordan, 11 Feb 2025

In this presentation, we consider a random map $x \mapsto F(x, \omega)$ and a random variable $\Theta(\omega)$. We denote by $F^N(x, \omega)$ and $\Theta^N(\omega)$ their approximations for some convergence parameter $N \to +\infty$. The questions to which we will provide clear and definitive answers are the following,

- Under which assumptions does the compound approximation $F^N(\Theta^N(\omega), \omega)$ converges in $\mathbf{L}_p$ to the compound map $w \to F(\Theta(\omega), \omega)$.
- What is the convergence rate and how does it depend on those related to the approximations F^N to F and Θ^N to Θ?

It is easy to guess that the analysis would be straightforward if (F, F^N) were independent of (Θ, Θ^N), by using a conditioning argument. On the contrary, here our aim is to allow arbitrary dependencies and study the strong convergence in this general setting (convergence in Lp-norms).

I give a number of applications including the resolution of some stochastic PDEs by the method of stochastic characteristics in a continuous framework and then in a framework with jumps with non-finite Lévy measures.
I also consider some approximations of stochastic processes (possibly non semi-martingales) at random times (possibly non stopping times). Examples include Brownian local times at random points, Fractional Brownian motions or diffusion processes at Brownian time.
Finally, an application to free up computer memory during large simulations and an application to learning an agent's preferences in the Human-Machine System framework are given.