

Alfons Ooms

Title and abstract

"The polynomiality of the Poisson center of a Lie algebra and Dixmier's fourth problem"

Abstract.

Let L be a finite dimensional Lie algebra over an algebraically closed field k of characteristic zero and let $S(L)$ be its symmetric algebra, equipped with its natural Poisson structure. We collect necessary and also some sufficient conditions for the Poisson center of $S(L)$ to be polynomial. This occurs for instance if L is quadratic of index 2 with $[L, L] \neq L$ and also if L is nilpotent of index at most 2. The converse holds for filiform Lie algebras of type L_n , Q_n , R_n and W_n .

Dixmier's problem, which is related to this, can be reduced to that of its canonical truncation. Moreover, Dixmier's statement holds for all Lie algebras of dimension at most 8. The nonsolvable ones among them possess a polynomial Poisson center and semi-center.