Workshop FBP 2017 :

Mathematical problems with subdifferential condition

20 – 21 juin 2017

<u>Résumés des conférences</u>

L^p-Theory for the Stokes and Navier-Stokes Equations with Different Boundary Conditions

Chérif AMROUCHE, Université de PAU et des Pays de l'Adour

Abstract. We consider here elliptical systems as Stokes and Navier-Stokes problems in a bounded domain, eventually multiply connected, whose boundary consists of multi-connected components. We investigate the solvability in L^p theory, with $1 < p < \infty$, under the non standard boundary conditions

$$\boldsymbol{u} \cdot \boldsymbol{n} = g, \quad \mathbf{curl}\, \boldsymbol{u} \times \boldsymbol{n} = \boldsymbol{h} \quad \text{or} \quad \boldsymbol{u} \times \boldsymbol{n} = \boldsymbol{g}, \quad \pi = \pi_* \quad \text{on}\, \Gamma.$$

We consider also the case of Navier boundary conditions:

$$\boldsymbol{u} \cdot \boldsymbol{n} = g \quad \text{and} \quad 2\left[\mathbf{D}(\boldsymbol{u})\boldsymbol{n}\right]_{\boldsymbol{\tau}} + \alpha \boldsymbol{u}_{\boldsymbol{\tau}} = \boldsymbol{h} \quad \text{on}\, \Gamma$$

where α is a friction coefficient and $\mathbf{D}(\boldsymbol{u}) = \frac{1}{2}(\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^{\top})$ is the stress tenseur. The main ingredients for this solvability are given by the Inf-Sup conditions, some Sobolev's inequalities for vector fields and the theory of vector potentials satisfying

$$\boldsymbol{\psi} \cdot \boldsymbol{n} = 0, \quad \text{or} \quad \boldsymbol{\psi} \times \boldsymbol{n} = \boldsymbol{0} \quad \text{on}\, \Gamma.$$

Those inequalities play a fundamental key and are obtained thanks to Calderon-Zygmund inequalities and integral representations. In the study of ellpitical problems, we consider both generalized solutions and strong solutions that very weak solutions.

In a second part, we will consider the nonstationary case for the Stokes equations.

References.

(1) C. Amrouche, C. Bernardi, M. Dauge, V. Girault, *Vector potentials in three-dimensional non-smooth domains*, M2AS, **21**, 1998, 823–864.

(2) C. Amrouche, A. Rejaiba, *L^p-theory for Stokes and Navier-Stokes equations with Navier boundary conditions*, Journal of Diff. Eq., **256**, 2014, 1515–1547.

(3) C. Amrouche, N.E.H. Seloula, *Theory for vector potentials and Sobolev's inequalities for vector fields. Application to the Stokes equations with pressure boundary condition*, Math. Mod. Meth. Appl. Sc., **23-1**, 2013, 37–92.

(4) J. Bolik, W. Von. Wahl, *Estimating ∇u in terms of* $\operatorname{div} u$, $\mathbf{curl}\, u$ *either* (ν, u) *and* $(\nu \times u)$ *and the topology*, Math. Meth. Appl. Sci., **20**, 1997, 737–744.

INERTIAL DYNAMICAL SYSTEMS WITH VANISHING DAMPING, AND FAST ALGORITHMS FOR NONSMOOTH CONVEX OPTIMIZATION

HEDY ATTOUCH

INSTITUT MONTPELLIÉRAIN ALEXANDER GROTHENDIECK, UMR 5149 CNRS,
UNIVERSITÉ MONTPELLIER 2, FRANCE

Abstract. Large scale optimization problems naturally appear in the modeling of many scientific and engineering situations. To meet the challenges posed by these issues, in recent years, considerable efforts have been devoted to the study of first-order splitting algorithms. The *forward-backward algorithm*, (also called the *proximal-gradient* algorithm) which is one of the most important, is a powerful tool for solving optimization problems with a *additively separable* and *smooth* plus *nonsmooth* structure. It is a natural extension of the gradient-projection algorithm. In the convex setting, a simple but ingenious acceleration scheme developed by Nesterov, and Beck-Teboulle improves the theoretical rate of convergence for the function values, in the worst case, from the standard $\mathcal{O}(k^{-1})$ down to $\mathcal{O}(k^{-2})$. In this lecture, we show that the rate of convergence of a slight variant of this accelerated forward-backward method, which produces *convergent* sequences, is actually $o(k^{-2})$, rather than $\mathcal{O}(k^{-2})$. Our arguments are based on the connection between this algorithm and a second-order differential inclusion with vanishing damping, recently introduced by Su, Boyd and Candès. Linking algorithms with dynamical systems provide connections with unilateral mechanics, control, PDE's and a valuable guide for the proofs. The key point of the mathematical analysis is the introduction of energy-like Lyapunov functions, with adapted scaling. We show that the introduction of additional geometric assumptions on the data (strong convexity of the objective function, Hessian-driven damping) leads to even better convergence rates. Finally, we consider the hierarchical multi-objective problem which consists in finding by rapid methods the solution with minimum norm of a convex minimization problem. To this end, we introduce into the dynamics and algorithms a Tikhonov regularization term with vanishing coefficient. Applications are given in sparse optimization for signal/imaging processing, and inverse problems.

References

[1] H. Attouch, A. Cabot, *Asymptotic stabilization of inertial gradient dynamics with time-dependent viscosity*, to appear in J. Differential Equations. 2017. HAL-01453108.

[2] H. Attouch, Z. Chbani, J. Peypouquet, P. Redont, *Fast convergence of inertial dynamics and algorithms with asymptotic vanishing damping*, to appear in Math. Program. DOI: 10.1007/s10107-016-0992-8. Published online 24 march 2016.

[3] H. Attouch, J. Peypouquet, *The rate of convergence of Nesterov's accelerated forward-backward method is actually faster than* $\frac{1}{k^2}$, SIAM J. Optim., 26 (2016), No. 3, pp. 1824–1834.

[4] A. Beck, M. Teboulle, *A fast iterative shrinkage-thresholding algorithm for linear inverse problems*, SIAM J. Imaging Sci., 2 (2009), No. 1, pp. 183–202.

[5] A. Chambolle, C. Dossal, *On the convergence of the iterates of the Fast Iterative Shrinkage/Thresholding Algorithm*, Journal of Optimization Theory and Applications, 166 (2015), pp. 968–982.

[6] Y. Nesterov, *Introductory lectures on convex optimization: A basic course*, volume 87 of Applied Optimization. Kluwer Academic Publishers, Boston, MA, 2004.

[7] W. Su, S. Boyd, E.J. Candès, *A Differential Equation for Modeling Nesterov's Accelerated Gradient Method: Theory and Insights*, Journal of Machine Learning Research, 17 (2016), pp. 1–43.

ADAPTATION DE LA METHODE DE NITSCHE AUX PROBLEMES DE CONTACT AVEC OU SANS FROTTEMENT EN ELASTODYNAMIQUE

(Avec Patrick HILD et Yves RENARD)

Franz CHOULY,

Université de Franche-Comté

Résumé : Dans cette présentation, on s'intéresse au problème de contact dynamique en élasticité avec ou sans frottement. On propose une adaptation de la méthode de Nitsche pour la semi-discrétisation en espace par éléments finis du problème. Les résultats présentés concernent l'existence et l'unicité des problèmes semi- et totalement discrétisés, la stabilité des schémas d'évolution en temps, et les simulations correspondantes.

UNE CLASSE D'INEQUATIONS VARIATIONNELLES IMPLICITES ET APPLICATIONS A DES PROBLEMES QUASISTATIQUES DE CONTACT

Marius COCOU

Université d'Aix-Marseille

Résumé : Ce travail concerne l'analyse d'une inéquation variationnelle d'évolution qui constitue une généralisation de plusieurs problèmes quasistatiques de contact avec frottement local de Coulomb en élasticité, dans l'hypothèse des petites perturbations mais avec une loi constitutive qui peut être non linéaire.
Dans la première partie, on montre l'existence d'une solution de cette inéquation par une technique incrémentale qui consiste à étudier une suite de problèmes discrets en utilisant un théorème de point fixe, plusieurs estimations pour les solutions discrètes ainsi que certains résultats de compacité.
Dans la deuxième partie, les résultats abstraits obtenus précédemment permettent l'étude de quelques problèmes quasistatiques avec une condition de contact unilatéral relaxée et frottement local de Coulomb, en particulier pour un milieu élastique linéaire ainsi que pour un corps avec une loi constitutive non linéaire, comme, par exemple, celle de Hencky.

A class of Variational-hemivariational Inequalities in Contact Mechanics

Mircea Sofonea

University of Perpignan Via Domitia, France
sofonea@univ-perp.fr

Keywords : contact problem, unilateral constraints, variational-hemivariational inequality, convergence results, optimal control, numerical simulations.

We start this lecture with a one-dimensional mathematical model which describes the equilibrium of an elastic rod in unilateral contact with a foundation, under the action of a body force. The weak formulation of this problem is in a form of an elliptic variational-hemivariational inequality for the displacement field, governed by a nonlinear operator, a convex set of constraints and two nondifferentiable functionals. Based on this example, we consider an abstract class of variational-hemivariational inequalities in reflexive Banach spaces, for which we present existence, uniqueness and convergence results. Then, we formulate two optimal control problems for which we prove the existence of optimal pairs, together with some additional convergence results. We illustrate the use of these abstract results in the study of various models of contact. For part of the models we also present numerical simulations which represent a numerical validation of our theoretical results.